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The unified equation applied to API 15HR

1 Introduction – This exercise illustrates the application of the unified equation to the API 15HR long-term cyclic requirement for composite pipes. The requirement is "no pipe failure after N = 10^9 cycles under nominal pressure and a strain ratio R = 0.9".

The problem will be addressed from the perspectives of failure by burst and by weep.

2 The global strains – We start with an estimation of the strains in the global "x" (axial) and "y" (hoop) directions of the pipe. From these we calculate the local strains in the principal directions of the UD plies, that is, the strains transverse to the fibers (direction 2) and in the fiber direction (direction 1).

The strain transverse to the fibers controls the weep failure. The strain in the fiber direction controls the long-term burst failure. The pipe is assumed to operate under the conditions stipulated in the API 15HR requirement stated in the introduction.

We proceed to calculate the global strains. We start with the relationship that exists between the hoop and axial global strains for angle-ply ± 55 pipes under a 2:1 pressure loading. This relationship is derived like follows:

The global stresses in closed cylinders under internal pressure are related to the global strains by the relationship.

$$[\sigma]=[A]\times[\varepsilon]$$

The stiffness matrix [A] for ± 55 angle-ply laminates with a 70% glass loading is

$$[A]=\begin{bmatrix}132470 & 92200 & 0\\ 92200 & 235070 & 0\\ 0 & 0 & 101220\end{bmatrix}$$

Entering this in the above equation we have

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix}=\begin{bmatrix}132470 & 92200 & 0\\ 92200 & 235070 & 0\\ 0 & 0 & 101220\end{bmatrix}\times\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix} \qquad (1)$$

For a 2:1 pressure loading and no external torque, as required by API, equation (1) reduces to

$$\begin{bmatrix}\sigma_x\\ 2\sigma_x\\ 0\end{bmatrix}=\begin{bmatrix}132470 & 92200 & 0\\ 92200 & 235070 & 0\\ 0 & 0 & 101220\end{bmatrix}\times\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix} \qquad (2)$$

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Expanding the above we obtain

$$\varepsilon_y = (3.41)\varepsilon_x \qquad (3A)$$

$$\gamma_{xy} = 0 \qquad (3B)$$

Equation (3A) indicates that the hoop strains are 3.41 times the axial strains in ± 55 angle-ply pipes under 2:1 pressure loading. Equation (3B) indicates absence of shear strains in the global reference frame.

The maximum allowable long-term hoop strain per API 15HR is obtained from the weep regression equation under static loading at 65C. In the following analysis we make use of one of the many regression lines available in the literature. Assuming a 20 years lifetime we have

$$\log(LTHS) = -0.01 - 0.06\log(20 \times 365 \times 24) \qquad (4)$$

$$LTHS = 0.48\%$$

The above static long-term hoop strain – LTHS – has a 50% probability of weeping the pipe after 20 years of continuous operation. The interpretation is like follows:

"A pipe that is filled with water and subjected to an internal pressure that produces a hoop static tensile strain of 0.48% will develop small cracks that allow the passage of water. It takes the water 20 years to travel through the cracks. After 20 years under sustained pressure, the water molecules traverse the cracks in the wall of the pipe and finally appear as droplets on the outside surface. This interpretation ignores the effect of the wall thickness".

The cracks that are caused by the hoop strain LTHS = 0.48% are very small indeed, since it takes 20 years for the water to travel the wall thickness. The reader is reminded that API 15HR assumes the pipes operating under static loadings and carrying products that do not corrode the resin. The cracks under such conditions are stationary, meaning they do not grow with time. The regression equation used to estimate the LTHS does not result from deterioration of the pipe. It simply measures the time taken by the water to travel through the stationary cracks in the wall. The regression line given by equation (4) estimates the travel time, and does not imply any long-term deterioration in the pipe

From API 15HR the allowable static long-term hoop strain for a lifetime of 20 years is

$$\varepsilon_y = f_1 f_2 (LTHS) = 0.85 \times 0.67 \times 0.48 = 0.27\%$$

Entering the above in equation (3) we obtain the corresponding static axial strain for a 2:1 loading.

$$\varepsilon_x = \frac{0.27}{3.41} = 0.08\%$$

These are the global strains that act in the axial and in the hoop directions of the pipe under nominal operating conditions. We next rotate these global strains to the principal ply directions.

3 The local ply strains – The strains in the local ply directions 1 – 2 are obtained by rotating the strains in the global system x – y. The rotation matrix for ± 55 plies is

$$[T]=\begin{bmatrix} 0.33 & 0.67 & \pm 0.94 \\ 0.67 & 0.33 & \mp 0.94 \\ \mp 0.47 & \pm 0.47 & -0.34 \end{bmatrix}$$

The strains in the principal ply directions 1 and 2 are

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \frac{1}{2}\gamma_{12} \end{bmatrix}=\begin{bmatrix} 0.33 & 0.67 & \pm 0.94 \\ 0.67 & 0.33 & \mp 0.94 \\ \mp 0.47 & \pm 0.47 & -0.34 \end{bmatrix}\times\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \qquad (5)$$

Entering the calculated global strains in the above we have

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \frac{1}{2}\gamma_{12} \end{bmatrix}=\begin{bmatrix} 0.33 & 0.67 & \pm 0.94 \\ 0.67 & 0.33 & \mp 0.94 \\ \mp 0.47 & \pm 0.47 & -0.34 \end{bmatrix}\times\begin{bmatrix} 0.08 \\ 0.27 \\ 0 \end{bmatrix}$$

$\varepsilon_1 = 0.18\%$ (Static strain in the fiber direction)

$\varepsilon_2 = 0.14\%$ (Static strain transverse to the fibers)

$\gamma_{12} = \pm 0.18\%$ (Shear strain referred to the local ply system)

The above are the static strains in the local ply directions 1 – 2 for ± 55 pipes under the API 15HR nominal operating conditions. They were calculated assuming the pipe operating under rated pressure.

4 Burst rupture – The long-term burst failure is controlled by the rupture of the glass fibers. API 15HR requires that we check the glass fibers for rupture under a cyclic load of N = 10^9 cycles at R = 0.9 superimposed on nominal static working conditions.

The unified equation predicts the long-term burst failure of pipes operating under the simultaneous action of static and cyclic tensile loads. The unified equation for strains acting in the ply (fiber) direction 1 is

$$\left(\frac{\varepsilon \times SF}{S_S}\right)^{\frac{1}{G_S}}+\left(\frac{\Delta\varepsilon \times SF}{S_C}\right)^{\frac{1}{G_C}}+\left(\frac{\varepsilon \times \Delta\varepsilon \times SF^2}{S_S \times S_C}\right)^{\frac{1}{G_{SC}}}=1{,}0 \qquad (6)$$

Where

$\epsilon = \epsilon_1 = 0.18\%$ is the tensile static strain in the fiber direction (direction 1)
$\Delta\epsilon = \Delta\epsilon_1 = 0.018\%$ (for R = 0.9) is the cyclic tensile strain range in the fiber direction
Ss = is the long-term (20 years) static strength of the glass fibers in the 1 direction
Sc = is the long-term cyclic strength of the glass fibers in the 1 direction
Gs = 0.077 (Mark Greenwood, boron-free glass)
Gs = 0.130 (Mark Greenwood, E glass)
Gc = 0.089 (John Mandell and Guangxu Wei, any glass)
Gsc = 7.0 (From Goodman's constant life diagrams)

The long-term static and cyclic strengths for the UD plies in the fiber direction are obtained from the appropriate regression lines.

The long-term static strengths for both E and boron-free glass in the 1 direction are

$\log(S_S) = 0.400 - 0.077\log(20 \times 365 \times 24)$ (Mark Greenwood for boron-free glass)

$S_S = 0.99\%$ (for boron-free glass)

$\log(S_S) = 0.347 - 0.130\log(20 \times 365 \times 24)$ (Mark Greenwood for E glass)

$S_S = 0.46\%$ (for E glass)

The long-term cyclic strength in the 1 direction is

$\log(S_C) = 0.519 - 0.089\log(N)$ (Guangxu Wei, for both E and boron-free glasses)

According to API 15HR, the number of cycles in 20 years is $N = 10^9$.

$\log(S_C) = 0.519 - 0.089\log(10^9)$

The long-term cyclic strength in the 1 direction is

$S_C = 0.522\%$ (Long-term cyclic strength for both boron-free and E glass)

Entering these values in the unified equation we have

For pipes made of boron-free glass:

$$\left(\frac{0.18 \times SF}{0.99}\right)^{\frac{1}{0.077}} + \left(\frac{0.018 \times SF}{0{,}522}\right)^{\frac{1}{0.089}} + \left(\frac{0.18 \times 0.018 \times SF^2}{0.99 \times 0{,}522}\right)^{\frac{1}{7}} = 1.0$$

SF = 5.0

The Unified Equation predicts that pipes made of boron-free glass under the combined action of cyclic and static loads as mandated in API 15HR have a long-term (20 years) safety factor SF = 5,0 against burst rupture.

For pipes made of E glass:

$$\left(\frac{0.18\times SF}{0.46}\right)^{\frac{1}{0.130}}+\left(\frac{0.018\times SF}{0,522}\right)^{\frac{1}{0.089}}+\left(\frac{0.18\times 0.018\times SF^{2}}{0.46\times 0,522}\right)^{\frac{1}{7}}=1.0$$

SF = 2.2

The foregoing analysis indicates that pipes made of either boron-free or regular E glass meet the API 15HR requirements against burst failure under the combined action of static + cyclic loading. Also, the analysis establishes the superior performance of boron-free glass (SF = 5.0) versus regular E glass (SF = 2.2).

The above is valid for long-term burst rupture. We next address the weep mode of failure.

5 Weep failure – The weep mode of failure results from water seeping through cracks that form when the fibers separate from the resin. This separation comes from glass–resin debonds as well as from cracks that form in the resin matrix. There are many micrographs in the literature showing the cracks forming along the glass fibers. Three facts seem to be undisputed:

- Weeping results from the passage of water through cracks that form parallel to the fibers.
- The cracks that initiate as fiber-resin debonds grow under cyclic loads along the fibers and also in the direction of the ply thickness. Their growth is controlled by the fiber-resin interface as well as by the resin matrix.
- The cracks only grow under cyclic strain. Static strains do not grow cracks.

Understanding how the cracks form and grow is very important to the study of the weep failure of composite pipes. We elaborate on this.

- The cracks under static tensile loads are stationary and do not grow. The opening, length and density of static cracks are fixed and do not change with time.
- Under cyclic tensile loads the cracks increase in length and density (number of cracks per unit volume) but do not increase in opening. The opening of the cracks (static or cyclic) is controlled by the pipe stiffness, which may be assumed constant throughout the process.

It is important that we understand this.

- Under cyclic loads the crack openings remain fixed, as determined by the pipe stiffness and the pressure, while their lengths and density grow with the number of cycles.
- Under static loads the cracks are stationary and do not grow in number, in length or in opening.
- As a consequence, the cracks that coalesce to form the pathway leading to weep failure are wider, fewer and shorter under static loads, than they are under cyclic loads.

In the process of weeping, the water molecules move along many narrow and long cracks. Starting from the inner surface of the pipe, the molecules move along cracks in the innermost + 55 ply until they cross with similar cracks in the – 55 ply immediately above. At this crossing point the molecules migrate from the inner ply to the outer ply and proceeds this way, from one ply to the next, until they weep out. The cracks are narrow, the pathways are tortuous and the travel distances are large. The time taken by the water to move from the inner ply to the outer ply may be very long. It is obvious that the time to weep depends on the pipe thickness. Other things being equal, the pipes having larger wall thickness will have longer weep times and flatter regression lines.

The weep time is not a fundamental material property. For one thing it depends on pipe thickness. For another, it depends on whether the cracks developed from static or from cyclic loads, that is, on the R ratio. One very simple way to obtain high LTHS in the ASTM D 2992 test protocol is by increasing the wall thickness of the test specimens. The LTHS (or classical HDB) is therefore not a fundamental material property and should not be used in pipe design. The fundamental pipe parameter controlling the weep process is the critical strain transverse to the fibers. This critical transverse strain is a fundamental material property that should be used in design. This critical transverse tensile strain is known as threshold weep strain.

The threshold weep strain will eventually replace the classical HDB in the design of composite pipes. This day, however, has not come yet.

Note: The threshold weep strain for static loadings, as well as the resistance to weep under cyclic loadings, depends on the base resin as well as on the fiber sizing. Over the years, the fiber sizing chemistry has been optimized for use in epoxy resins. A similar effort is required for polyesters and for vinyl esters.

The threshold weep strain is a “safe strain” for protection against weeping from static loads. When the pipe is static loaded below the weep strain, the cracks that develop are either too few or too short and do not coalesce to form pathways for the passage of water. And since static cracks do not grow, weeping never happens.

As it happens, however, the threshold weep strain is not a safe strain for weeping from cyclic loads. This is because under cyclic loads the cracks will grow and eventually coalesce. This happens regardless of crack size and tensile strain. Crack growth occurs under cyclic loadings even for strains less than the weep threshold.

According to Guangxu Wei, the static weep threshold for tensile loads in the 2 direction of polyester UD plies is $\varepsilon_2 = 0.25\%$. Acoustic emission studies conducted at the University of Liverpool have suggested a weep threshold of $\varepsilon_2 = 0.20\%$.

In this paper we will use $\varepsilon_2 = 0.25\%$ as the static weep threshold. This value comes from tests performed on polyester plies and should be higher for epoxy or for vinyl ester resins.

Returning to our problem of the API requirement under cyclic load, we note that the safety factor against the peak strain exceeding the weep threshold is

$$SF = \frac{(threshold \quad strain)}{\varepsilon + \frac{\Delta\varepsilon}{2}} \qquad (7)$$

For API 15HR pipes operating under rated conditions, the static strain in the 2 direction was calculated earlier as $\epsilon_2 = 0.14\%$. The cyclic component of the tensile strain in the 2 direction at R = 0.9, is $\Delta\epsilon_2 = 0.1 x 0.14\% = 0.014\%$. Therefore, the safety factor for API 15HR pipes against peak strains is

$$SF = \frac{0{,}25}{0{,}14 + \frac{0{,}014}{2}} = 1{,}63 \qquad (8)$$

There is no risk of the pipe ever weeping under the peak strain from the combined action of static and cyclic strains specified in API 15HR. So, we are safe against weep failure as far as the magnitude of the combined loading is concerned.

However, the API 15HR specifies a cyclic loading. And we know that cyclic loads grow cracks that may eventually weep the pipe. We next check the pipe for cyclic weep failure. Weep failure under cyclic loads requires the fulfillment of two conditions.

- First, the initial small debond cracks must grow under the cyclic strain until they coalesce to form the pathway. Let us call this the coalescence time.
- Second, the water must travel the pathway thus formed. Let us call this the travel time.

The weep time under cyclic loading is obtained by adding two times.

$$[weep \quad time] = [coalescence \quad time] + [travel \quad time]$$

The coalescence time will be assumed equal to the ply rupture time in the 2 direction. This is a conservative approach. The time to ply rupture is computed by the unified equation. For transverse tensile rupture in the 2 direction the unified equation is

$$\left(\frac{\Delta\varepsilon \times SF}{S_C}\right)^{\frac{1}{0,040}} + \left(\frac{\varepsilon \times \Delta\varepsilon \times SF^2}{S_S \times S_C}\right)^{\frac{1}{G_{SC}}} = 1,0 \qquad (9)$$

Where

$\Delta\epsilon$ = 0.014% (Cyclic component in the 2 direction. R = 0.9)
S_c = Long-term cyclic strength in the 2 direction assuming a purely cyclic loading and N = 10^9 cycles.
S_s = Long-term static strength in the 2 direction, 20 years
ϵ = 0.14% (Static component in the 2 direction)
G_{sc} = Interaction parameter for the strain ratio R = 0.9

The long-term cyclic strength S_C, that is, the strain that initiates the coalescence process in the 2 direction, is assumed to be the strain that fully cracks the UD ply. The regression cyclic line for this is provided by Guangxu Wei as

$$\log(S_c) = \log 0,25\% - 0,040 \log N$$

In the above equation 0.25% is the short-term tensile strain that ruptures the ply in the transverse direction. In our case the API requirement calls for N = 10^9 cycles. The long-term cyclic strength S_c computed from the above equation is

$$\log(S_c) = \log 0,25\% - 0,040 \times \log 10^9$$

S_c = 0.109%

The long-term static strength in the transverse direction is the same as the short-term strength, because there is no strain-corrosion in the direction transverse to the fibers. Therefore

S_S = 0.25%

Entering the above in the unified equation we obtain

$$\left(\frac{0,014 \times SF}{0,109}\right)^{\frac{1}{0,040}} + \left(\frac{0,14 \times 0,014 \times SF^2}{0,25 \times 0,109}\right)^{\frac{1}{G_{SC}}} = 1,0 \qquad (10)$$

The unified equation cannot be used to compute the safety factor SF in this case, since the interaction parameter G_{sc} is not known for this particular loading (R = 0.9). We can, however, solve equation (9) using the conservative value G_{sc} = 40 000 which holds for N = 10^9 cycles and R = 0.5.

Entering G_{sc} = 40 000 in equation (10) we obtain

SF = 3.0

The safety factor SF = 3.0 indicates that the UD plies in the pipe will not crack under the specified loading in 20 years. The absence of cracking means no coalescence, no pathways and no weeping. We can be sure that for a desired lifetime of 20 years the pipes under the cyclic loading prescribed in API 15HR will not crack and therefore will not weep.

This complete our analysis of the API 15HR cyclic problem.

6 The travel time. The preceding section indicated that the weep time is obtained by adding the coalescence time and the travel time. We have shown that the coalescence time under the loading conditions prescribed by API 15HR is higher than 20 years, which makes the travel time irrelevant.

However, had the coalescence time been less than 20 years, the travel time would not require computation since the water most likely would have soaked up the cracks as they formed in the long cracking time. The travel time would then be very short and could be ignored.

This is a conservative approach. The reasoning is like follows: the pipe is considered as failed from the moment the cracks have coalesced, even in the absence of weeping. The real problem is seen as the coalescence, not the weeping.

8 Conclusion – The analysis conducted using the unified equation reveals that:

1. The long-term cyclic requirement imposed by API 15 HR ($N = 10^9$ cycles in 20 years with R = 0.9) is easily met for rupture (burst) failure.
2. The boron-free glass is superior to the regular E glass in rupture failure.
3. The unified equation could not be used to do the weep analysis for lack of specific data to estimate the interaction coefficient Gsc for R = 0.9 and $N = 10^9$
4. However, using conservative data for R = 0.5 and $N = 10^9$, we have determined that the pipes will not weep in 20 years under the combined static + cyclic loading specified in API 15HR.

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