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A unified equation to predict the rupture life of composites under cyclic and static loadings

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1 Abstract – The long-term rupture of composite laminates under the separate action of static and cyclic loadings can be predicted by well established regression equations. These equations are good predictors of failure when the static and the cyclic loads act individually. The difficulty arises when the two loads act at the same time, and the designer has to make use of cumbersome constant life diagrams – also known as Goodman diagrams. The industry needs an accurate unified equation capable of predicting the rupture life of structures under combined static + cyclic loadings.

This paper introduces a unified equation that predicts the rupture life of composite pipes subjected to simultaneous static and cyclic loadings.

2 Two lives - The durability of composite pipes can be addressed from two points of view, each leading to a specific "life".

1 – The **service life** is concerned with the fitness for use of the pipe. In industrial pipes, carrying chemicals, the service life ends when the aggressive products fully penetrate the corrosion barrier and reach the structural plies. The penetration of the corrosion barrier exposes the structural plies to the aggressive chemical and this may lead to rapid failure. When that happens the pipe is ruled out as unfit for service and we define the service life as the time to full penetration of the corrosion barrier.
2 – The **structural life** is concerned with the inability of the pipe to carry loads. The structural life comprises three modes of failure.

- The first mode is burst, defined as separation of the pipe in distinct pieces under long-term internal pressure. The burst failure may be caused by cyclic fatigue or by the combined action of static loads and the water attack on the fibers.
- The second mode of failure is strain-corrosion, defined as rupture of the pipe under long-term bending loads while immersed in corrosive chemicals.
- The third mode of structural failure is weep, defined as the passage of fluids through the pipe wall. The weep failure is time independent and should not be included in the long-term mode category.

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These three modes of structural failure are driven by different mechanisms. Burst results from fiber rupture. Weep results from cracks that develop when the fibers debond from the resin. And the strain-corrosion failure occurs under bending loads and in the presence of corrosive chemicals. For details on these modes of failure, see references 2, 3 and 4.

The unified equation introduced in this paper predicts the weep or burst lives of composite pipes under the simultaneous action of static and cyclic loadings. The strain-corrosion rupture cannot be predicted by the unified equation.

3 Two aggressive chemicals - We define as aggressive any chemical that lessens the performance of the pipe. The aggressive chemicals can be grouped in two categories.

1 - The ***non-penetrating*** chemicals interact with the plies next to the pipe's inner surface. Most chemical products found in industrial applications fall in this category. The low penetrating power of the chemical products limits their attack to the corrosion barrier. We assume that the non-penetrating chemicals affect the service life and have no effect on the structural life.

2 - The ***penetrating*** chemicals reach all plies. Chemicals like solvents, especially water, fall in this category. These chemicals can interact with the structural plies and may affect the structural life.

4 A special penetrating chemical – The corrosion of metals is a good example of a non-penetrating process. The impermeable metallic lattice prevents the penetration of chemicals that would corrode the metal from within. As a result the metals are corroded from the surface, with a very well defined corrosion front. The material that is corroded ceases to play any structural role, while the remaining metal keeps its properties intact. Metallic structures lose load capacity in corrosive environments, not because they lose mechanical properties, but because they lose thickness.

A similar process occurs with composites in non-penetrating chemicals. The plies that are penetrated lose mechanical properties and, as in the case of metals, reduce the load carrying capacity of the structure. The non penetrated plies, again like in metals, keep their original properties intact. This statement is not obvious. In fact, the conventional interpretation for the attack of chemicals on composites takes exactly the opposite view. The ASTM C581 test method assumes that the chemicals (a) fully penetrate and (b) deteriorate the properties of the entire laminate while (c) maintaining the structural thickness unchanged. As the reader can see, these two views are in direct opposition.

Unlike the non-penetrating chemicals, the solvents can fully penetrate the laminate. Fortunately the solvents are not reactive and do not attack the resin or the fibers. The harm that they do is mostly mechanical, caused by swelling. The laminate swelling can range from moderate and fully reversible, to excessive and irreversible. In some cases the swelling is so intense as to crack or disintegrate the composite. The effect of solvents on the structural life, however, is not a major concern. Let us see why.

- The moderate swelling solvents cause damage that is (a) non-progressive and (b) can be evaluated within a short time of exposure.
- The aggressive solvents also tell their sad story within a short period of time.

The structural life of composites in solvents is a “go no-go” situation involving either full destruction or moderate deterioration that can be quantified and taken into account. Both the full destruction and the moderate deterioration can be ascertained by short-term immersion tests.

However, there is one special moderate swelling solvent that is reactive and therefore capable of causing progressive long-term damage. That special solvent is water. Water can be harmful in two ways. First, like all moderate solvents, it swells the laminate causing a small and reversible decrease on the HDT and on the mechanical properties. This effect is minor, since the water pick-up is small. The second effect involves a continuous and sustained attack on the glass fibers. The swelling effect of the water is minor and reversible. The slow and continued water attack on the glass fibers, however, is irreversible and cumulative. Eventually the fibers fail and cause the rupture the composite.

5 Two hypotheses – The foregoing discussion justifies the following hypotheses.

1st hypothesis - The non-penetrating chemicals interact with the inner plies and have no effect on the structural plies and the mechanical properties. The damage that they do is restricted to the corrosion barrier and their effect is limited to the service life. The non-penetrating chemicals have no effect on the mechanical properties or on the structural life.

2nd hypothesis - Water is the only agent that penetrates the entire laminate and degrades the mechanical properties. The long-term rupture of composite structures is caused by water in combination with static tensile strains.

The above hypotheses separate the service life – caused by chemicals – from the rupture life – caused by water. The knowledge of the service life is useful to schedule shutdowns for maintenance. The knowledge of the rupture life is useful in designing long-term infrastructure applications. The recognition that water is the only agent affecting the rupture life is a tremendous simplification to this otherwise intractable problem.

This paper will address the long-term rupture life of composites under the simultaneous action of cyclic and static loads. The weep life is addressed in reference 2. The service life is addressed in reference 1.

6 The rupture regression lines - This section discusses the significance of the regression lines used to assess the long-term rupture of composites. According to fracture mechanics, cracks under static loads are stationary and do not grow. Crack growth is observed only in cyclic (fatigue) loads. There is one exception to this rule, however. Crack growth can occur under static loads if the material is placed in corrosive environments. This condition is known as strain-corrosion. Strain-corrosion allows the growth of otherwise stationary cracks under static loads.

The long-term rupture from strain-corrosion results from the cracks increasing in size (under static strain) to a critical length. According to fracture mechanics the rate of crack growth in a material under strain-corrosion is governed by the Paris law which takes the form

$$\frac{da}{dt} = Y\left(\varepsilon \times \sqrt{\pi \times a}\right)^{Z} \qquad (1)$$

Where

"a" is the size of the crack.
"t" is the time
"Є" is the tensile strain.
"Y" is a constant dependent on geometry.
"Z" is a constant related to the rate of material deterioration.

Equation (1) is known as Paris law. It indicates that the rate of crack growth is controlled by the crack size "a" as well as by the applied static tensile strain "Є". The effect of the corrosive environment is accounted for by the exponent "Z". Equation (1) is valid for homogeneous materials, like glass fibers, that grow one large crack. Non-homogeneous materials, like composites, do not grow one large crack. Instead, they grow many small cracks that eventually coalesce and rupture the structure. That is the reason why the regression lines for composites link "tensile strain and time to rupture", instead of "crack size and time to rupture". The concept of crack size is good for homogeneous materials, like metals, but is not applicable to composites. The regression equation that predicts the rupture of composites under static loads is:

$$\log \varepsilon(t) = C_s - G_s \log(t) \qquad (2)$$

Where "Є(t)" is the static tensile strain and "t" is the time to rupture. The reader should note that equation (2) links strains (not crack size) with time. As we explained, this is because composites do not grow large cracks. The coefficients "C_s" and "G_s" are determined experimentally using regression methods. The regression lines predict the times to failure when the applied strains are known, and vice versa.

For a lifetime of "X" years, equation (2) would be written as

$$\log \varepsilon(X) = C_s - G_s \log(X) \qquad (2A)$$

Combining equations (2) and (2A), we obtain

$$\frac{X}{t} = \left(\frac{\varepsilon(t)}{\varepsilon(X)} \right)^{\frac{1}{G_S}} \qquad (2B)$$

In equation (2B) the strain "Є(t)" and the time "t" are pairs of rupture points. Given the time to rupture "t" we can immediately compute the strain "Є(t)" from equation (2). The strain "Є(X)" is the hoop tensile strain that rupture the pipe in X years. The parameter G_s reflects the resistance of the glass fibers to the attack by water. Equation (2B) is crucial in the derivation of the unified equation.

Equation (2) is valid for static tensile loads. The analog equation for the cyclic case is

$$\log \Delta\varepsilon(N) = C_c - G_c \log(N) \qquad (3)$$

Where "ΔЄ(N)" is the cyclic strain that ruptures the composite after "N" cycles. Note that in equation (3) the coefficients C and G have taken a subscript "c" to indicate the cyclic case. In full analogy with the static case we have, for a lifetime of X cycles

$$\log \Delta\varepsilon(X) = C_c - G_c \log(X) \qquad (3A)$$

$$\frac{X}{N} = \left(\frac{\Delta\varepsilon(N)}{\Delta\varepsilon(X)}\right)^{\frac{1}{G_C}} \qquad (3B)$$

Our objective is to combine equation (2B) and equation (3B) into one unified equation. To do this we need to define a time base that is applicable to both the static and the cyclic case. We must therefore define X and N for the cyclic case not in terms of number of cycles, but in terms of time. This is easily accomplished, as the reader will see in the numerical examples.

The parameters "C_s", "G_s", "C_c" and "G_c" are determined experimentally. They are known. Once these parameters are known, the regression lines (2) and (3) can be used to predict any pair of failure points.

The following definitions apply

Є(X) is the static strain that ruptures the pipe in X years
Δϵ(X) is the cyclic strain that ruptures the pipe in X years
X is the desired lifetime in years
X also represents the number of cycles in X years
Є(t) is the static strain that ruptures the pipe at the time t.
t is the time to rupture under the static strain Є(t)
N is the number of cycles to rupture under the cyclic strain Δϵ(N)

The strain that results in long-term (usually 50 years) rupture is known as the long-term strength. This paper will use the symbol "S" to represent the long-term strength. There are two types of "S", the cyclic S_C and the static S_S.

$\Delta\varepsilon(X) = S_c$ — Cyclic strength for a lifetime of X years

$\varepsilon(X) = S_s$ — Static strength for a lifetime of X years

Note: The rupture regression lines are determined on isolated plies. The tests performed on laminates composed of more than one ply are not valid. The regression lines should not be measured on water filled pipes as described in ASTM D2992 procedures A and B. The ASTM D2992 protocol is performed on pipes to generate weep regression lines, not rupture lines. In discussing pipe rupture, we are not interested in the weep lines. The regression lines of interest to the study of rupture failure are those developed by testing isolated plies to rupture, as done by Guangxu Wei (ref 5) and Mark Greenwood (ref. 6).

Using the new notation for the long-term strengths, equations (2B) and (3B) can be written as

$$\frac{X}{t} = \left(\frac{\varepsilon(t)}{S_S}\right)^{\frac{1}{G_S}} \qquad (2BA)$$

$$\frac{X}{N} = \left(\frac{\Delta\varepsilon(N)}{S_C}\right)^{\frac{1}{G_C}} \qquad (3BA)$$

Equations (2BA) and (3BA) hold for static and cyclic loads acting alone. The question is how to combine these two equations into one unified equation that would account for the effects of the cyclic and static loads acting simultaneously.

7 The unified equation - The unified equation is easy to derive. Equations (2BA) and (3BA) express the fractions of the total lifetime ascribed to each loading, taking as reference a desired lifetime of X years. To understand this, assume we set the desired lifetime to X = 50 years and have the following lives for the static and cyclic loadings acting alone:

- 500 years lifetime for a cyclic loading acting alone. In this situation the desired lifetime X = 50 years corresponds to 50/500 = 0.10 = 10% of the cyclic lifetime.
- 200 years lifetime for a static loading acting alone. By the same reasoning, the desired lifetime X = 50 years corresponds to 50/200 = 0.40 = 40% of the static lifetime.

The above tells us that neither the static nor the cyclic loading, if acting alone, fails the pipe in X = 50 years. The question that we ask is the following. Would the above loadings fail the pipe in X = 50 years if they acted simultaneously? According to the principle of superposition, when acting together for a period of 50 years, the two loadings should consume 40% + 10% = 50% of the pipe's lifetime. Therefore, after 50 years under the simultaneous action of the above two loadings, the pipe would still retain 50% of its life, and should not fail. The reader should note that we said the pipe retains 50% of its life, not 50% of its strength. The unified equation is about structural life, not about residual strengths.

Based on the above, the combined loading that ruptures the pipe in 50 years of continuous service is obtained when the two partial lifetimes add up to 100%, or

$$\left(\frac{50}{t}\right)_{static} + \left(\frac{50}{N}\right)_{cyclic} = 1{,}00$$

Which, in view of equations (2BA) and (3BA), is the same as

$$\left(\frac{\varepsilon}{S_s}\right)^{\frac{1}{G_S}} + \left(\frac{\Delta\varepsilon}{S_c}\right)^{\frac{1}{G_C}} = 1{,}00 \qquad (4)$$

In equation (4) "Gs" and "Gc" are the known slopes of the static and cyclic regression rupture lines. "Ss" and "Sc" are the known long-term strengths that rupture the critical ply when the cyclic and static loads act alone.

In real life applications the operating strains Є and ΔЄ (both known) are small and will not fail the pipe in the desired lifetime of X years. In order to fail the pipe, the pair of static and cyclic strains in equation (4) should be multiplied by a factor SF. Entering this SF factor in (4) we obtain

$$\left(\frac{\varepsilon \times SF}{S_s}\right)^{\frac{1}{G_S}} + \left(\frac{\Delta\varepsilon \times SF}{S_c}\right)^{\frac{1}{G_C}} = 1{,}00 \qquad \text{(4A)}$$

Equation (4A) calculates the factor SF that multiplies the pair of operating static and cyclic strains in order to rupture the critical ply (and the pipe) in the desired lifetime. The factor SF is interpreted as a long-term factor of safety when the cyclic and static loads act simultaneously.

Equation (4A) ignores the interaction between the cyclic and the static loads. The complete unified equation, that takes this interaction into account, is

$$\left(\frac{\varepsilon \times SF}{S_s}\right)^{\frac{1}{G_S}} + \left(\frac{\Delta\varepsilon \times SF}{S_c}\right)^{\frac{1}{G_C}} + \left(\frac{\varepsilon \times \Delta\varepsilon \times SF^2}{S_S \times S_C}\right)^{\frac{1}{G_{SC}}} = 1{,}00 \qquad \text{(5)}$$

Where the interaction coefficient Gsc is determined experimentally.

Equation (5) is the final form of the unified equation. It calculates the safety factor SF for any given combination of static and cyclic loading acting simultaneously for any desired period of time. The unified equation is not concerned with the residual strength of the composite. Instead, it gives the safety factor SF for the applied loading acting continuously over the desired lifetime of X years. The knowledge of the residual safety factor SF is better than the knowledge of the residual strength.

The unified equation is a neat, symmetrical and elegant mathematical expression. It can be easily extended to analyze the rupture failure of composites under the simultaneous action of cyclic and static loads in compression, tension and torsion both in the 1 and the 2 directions of the critical ply.

The analysis for the transverse 2 direction is of particular interest in the study of the weep mode of failure. For details, see ref. 4.

8 Using the unified equation – The following inputs are required to calculate the safety factor SF from equation (5).

The slope Gs of the static regression line
The slope Gc of the cyclic regression line
The interaction coefficient Gsc
The long-term static strength Ss of the critical ply
The long-term cyclic strength Sc of the critical ply

The long-term strengths and the slopes are obtained from the known static and cyclic regression lines. The only unknown parameter in equation (5) is the interaction Gsc, which can be determined from the values published in the constant life diagrams – also known as Goodman diagrams. Table 1 shows some suggested values for these parameters.

9 Worked example – In this worked example we will use equation (5) to calculate the safety factor SF against long-term rupture for underground hoop-chop composite pipes carrying water under the following conditions.

Hoop static tensile strain	$\epsilon = 0.25\%$
Hoop static bending strain	$\epsilon_b = 0.15\%$
Hoop cyclic tensile strain	$\Delta\epsilon = 0.15\%$
Desired lifetime	*X = 50 years*
Number of cycles in 50 years	*N(50) = 219.000.000 cycles*

The critical plies for hoop-chop pipes are the UD plies that are wound in the hoop direction. The problem will be solved for two situations:

- Pipes made of E glass
- Pipes made of boron-free glass.

Since this is a hoop-chop pipe, the critical UD plies have the same direction as the hoop strains and no strain rotation is required. For angle-ply laminates (for instance the ±55 degrees used in oil pipes) the hoop strains would be rotated to the fiber direction in order to apply the unified direction.

We are assuming that the pipe bursts when the UD plies fail. The bending and tensile strains have the same direction and can be added to give a total static tensile strain of 0.25% + 0.15% = 0.40% in the hoop direction.

Parameter	Value	Source
Gs	0,077 0,130	Mark Greenwood for UD plies of boron-free glass (ref. 6) Mark Greenwood for UD plies of E glass (ref. 6).
Gc	0,089	John Mandell (ref. 7) and Guangxu Wei (ref. 5)
Gsc	7000	Gsc varies with the number of cycles N and the strain ratio R. The value quoted here is based on data by Guangxu Wei for UD plies, $N = 10^6$ cycles and R = 0.5 (ref. 5).
Ss	0.92% 0.41%	Mark Greenwood for UD plies of boron-free glass and a lifetime X = 50 years (ref. 6) Mark Greenwood for UD plies of E glass and a lifetime X = 50 years (ref. 6).
Sc	NA	Sc depends on the number of cycles at X = 50 years and should be determined on a case by case basis from the applicable regression line (see equation 6).

Table 1
Inputs for the unified equation, assuming a lifetime of X = 50 years.

Table 1 provides the input data required by the unified equation (5) for a desired lifetime of X = 50 years. The X = 50 years cyclic strength, Sc, is calculated from equations (6) and (6A) (see ref. 1).

$$\log S_C = 0{,}519 - 0.089 \log N(50) \quad \text{(rupture in the 1 direction)} \qquad (6)$$

$$\log S_C = -0.699 - 0.04 \log N(50) \quad \text{(rupture in the 2 direction)} \qquad (6A)$$

We are interested in the failure of the pipe by rupture, that is, in the failure of the glass fibers in the 1 direction. For that reason the long-term cyclic strength of interest is obtained from equation (6). Equation (6A) would be used if we were analyzing the pipe for weep failure. This paper will not address the weep failure.

The factor of safety SF is a multiplier which brings the acting strains to the point of failure in X = 50 years. It is calculated from the unified equation (5).

Equation (5) will be solved for both E glass and boron-free glass.

9.1 - Boron-free glass – The long-term cyclic strength S_C for boron-free glass in the 1 direction is, from equation (6)

$$\log S_C = 0.519 - 0.089 \log 219 \times 10^6$$

Which gives S_C = 0.60%.

We now have all the necessary parameters to calculate the safety factor SF for pipes made of boron-free glass. The safety factor SF is obtained from equation (7)

$$\left(\frac{0.40 \times SF}{0.92}\right)^{\frac{1}{0{,}077}} + \left(\frac{0{,}15 \times SF}{0{,}60}\right)^{\frac{1}{0{,}089}} + \left(\frac{0.40 \times 0.15 \times SF^2}{0.92 \times 0{,}60}\right)^{\frac{1}{7000}} = 1{,}00 \qquad (7)$$

Which gives SF = 1.15. The meaning of this safety factor is like follows. The applied loading is not enough to fail the pipe in X = 50 years. For rupture to occur in 50 years the applied loading should be 1.15 times higher. Therefore SF = 1.15. Introducing this safety factor in (7) we can have an idea of the effect of each loading on the durability of the pipe.

$$\left(\frac{0.40 \times 1{,}15}{0.92}\right)^{\frac{1}{0{,}077}} + \left(\frac{0{,}15 \times 1{,}15}{0{,}77}\right)^{\frac{1}{0.089}} + \left(\frac{0.40 \times 0.15 \times 1{,}15^2}{0.92 \times 0{,}77}\right)^{\frac{1}{7000}} = 1{,}00$$

0.01% + 0.00% + 99.97% = 100%

This result is unexpected. The static and cyclic loadings per se have little effect on the durability of the pipe. The damage is caused by their combined effect, that is, by the interaction between the two loadings.

9.2 - E glass – The long-term cyclic strength S_C for E glass is 0.60%, the same as for boron-free glass. This is because the mechanism of cyclic fatigue is independent of the glass composition. The long-term static strength for E glass is (table 1) Ss = 0.41%. Entering these values in equation (5)

$$\left(\frac{0.40\times SF}{0.41}\right)^{\frac{1}{0,130}}+\left(\frac{0,15\times SF}{0,60}\right)^{\frac{1}{0.089}}+\left(\frac{0.40\times 0.15\times SF^2}{0.41\times 0,60}\right)^{\frac{1}{7000}}=1,00$$

Which gives SF = 0.50, meaning the pipe made of E glass bursts before completing 50 years in service. This example illustrates the superiority of boron-free glass vis-à-vis E glass. The higher hydrolytic stability of boron-free glass produces pipes with higher resistance to long-term burst failure.

A similar analysis can be performed for the 2 direction, to compute the weep life. This, however, is beyond the scope of the present paper. The interested reader may see ref. 8.

10 - Conclusion - We have derived a unified equation that analyzes the rupture life of composite pipes under the simultaneous action of static and cyclic loadings. This unified equation is expected to replace the cumbersome Goodman diagrams that have been used in such cases. The high value of the parameter Gsc indicates that the interaction between the static and cyclic loadings is the overriding factor in the determination of the rupture life of composites.

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Carlos Marques started his brilliant and short career as an engineer for Occidental Petroleum. Later experience includes 20 years with composites in industrial and sanitation applications. His last position was technical/commercial director for Polyplaster, a leading composites fabricator in Latin America. Carlos passed away in June of 2010, in his prime, at the early age of 52.

Appendix 1

The principle of superposition

Is the principle of superposition applicable in this case? Is this principle valid to combine the static and cyclic deteriorations in the same equation?

To be valid, the principle of superposition requires that the tensile and the cyclic modes operate under distinct mechanisms. And this is in fact the case. The crack growth under static loads is caused by strain-corrosion of the glass. And the crack growth under cyclic loading is caused by a non-chemical process.

The distinct mechanisms indicate that the static and cyclic modes of failure are unrelated and therefore amenable to superposition.