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General equation for ± 55 angle-ply oil pipes

Antonio Carvalho - IBCom

Introduction: The failure envelope proposed in the standard ISO 14692 is too cumbersome, artificial and inaccurate. This, in spite of the fact that the composites industry has, for a long time, been using the classical laminate theory to deal with problems such as the ones encountered in ISO 14692. This paper discusses the application of the general equation from classical laminate theory in the analysis of oil pipes. The general equation is more accurate and simpler to use than the failure envelope proposed in the ISO standard.

1 The general equation: The general equation for the analysis of composite circular cylinders under the simultaneous action of the fluid pressure, torque and axial load is

$$\begin{bmatrix} \frac{P\times\Phi}{4}+N_x \\ \frac{P\times\Phi}{2} \\ \frac{2\times T}{\pi\times\Phi^2} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \times \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \qquad (1)$$

Where

x is the axial direction
y is the hoop direction
P is the fluid pressure
Φ is the pipe diameter
N_x is the axial external force per unit length
T is the torque
[A] is the tensile stiffness matrix for the laminate
[Є] is the strain matrix referred to the global system

2 Angle-ply laminates: The pipes used in the oil and gas industry are made from balanced angle-ply laminates. For angle-ply balanced laminates equation (1) reduces to

$$\begin{bmatrix} \frac{P\times\Phi}{4}+N_x \\ \frac{P\times\Phi}{2} \\ \frac{2\times T}{\pi\times\Phi^2} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ 0 & 0 & A_{33} \end{bmatrix} \times \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \qquad (2)$$

The laminate stiffness matrix [A] depicted in equation (2) varies with the glass loading and the winding angle. For ± 55 degrees angle-ply laminates with 70% glass loading, this stiffness matrix is

$$[A]=\begin{bmatrix}132470 & 96220 & 0\\ 96220 & 235070 & 0\\ 0 & 0 & 101220\end{bmatrix}\times[t]$$

In the above, "t" is the total laminate thickness. The matrix [A] is expressed in kg⁄cm². Entering this matrix [A] in (2) we obtain

$$\begin{bmatrix}\frac{P\times\Phi}{4}+N_x\\ \frac{P\times\Phi}{2}\\ \frac{2\times T}{\pi\times\Phi^2}\end{bmatrix}=\begin{bmatrix}132470 & 96220 & 0\\ 96220 & 235070 & 0\\ 0 & 0 & 101220\end{bmatrix}\times[t]\times\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix} \qquad (3)$$

Dividing through by the laminate thickness "t" we obtain equation (3A), which is equation (3) expressed in terms of stresses

$$\begin{bmatrix}\frac{P\times\Phi}{4\times t}+\sigma_x\\ \frac{P\times\Phi}{2\times t}\\ \frac{2\times T}{\pi\times\Phi^2\times t}\end{bmatrix}=\begin{bmatrix}132470 & 96220 & 0\\ 96220 & 235070 & 0\\ 0 & 0 & 101220\end{bmatrix}\times\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix} \qquad (3A)$$

Equation (3A) links the pressure, torque and axial stress with the global axial, hoop and shear strains. The stiffness matrix in equation (3A) is valid for ± 55 angle-ply laminates with a glass loading of 70%.

Note: The stiffness matrix [A] is expressed in kg/cm². Therefore both the pressure and the stresses in equation (3A) should be expressed in kg/cm² as well.

3 Strain analysis: The analysis of composite pipes is best done using laminate failure strains, instead of laminate strengths. The reasons for the use of strains in place of stresses are:

1 – The failure strains are not affected by Poisson effects. The failure envelope for strains is rectangular, not elliptical as for strengths.
2 – Unlike strengths, the failure strains are basic lamina properties that do not vary with the laminate construction.

Equation (3A) solved for strains gives

$$\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix}=\begin{bmatrix}132470 & 96220 & 0\\ 96220 & 235070 & 0\\ 0 & 0 & 101220\end{bmatrix}^{-1}\times\begin{bmatrix}\frac{P\times\Phi}{4\times t}+\sigma_x\\ \frac{P\times\Phi}{2\times t}\\ \frac{2\times T}{\pi\times\Phi^2\times t}\end{bmatrix} \qquad (4)$$

Which, upon inversion of matrix [A] becomes

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} 1{,}07 & -0{,}44 & 0 \\ -0{,}44 & 0{,}61 & 0 \\ 0 & 0 & 0{,}99 \end{bmatrix} \times \left[10^{-5}\right] \times \begin{bmatrix} \frac{P \times \Phi}{4 \times t} + \sigma_x \\ \frac{P \times \Phi}{2 \times t} \\ \frac{2 \times T}{\pi \times \Phi^2 \times t} \end{bmatrix} \qquad (4A)$$

The strains in equation (4A) are referred to the global system "x – y" of the pipe, where "x" is the axial direction and "y" the hoop direction. The analysis of composite laminates requires the rotation of the global strains to the local 1 – 2 frames. We therefore rotate the strains from the global "x – y" system to the local "1 – 2" system, where "1" is in the fiber direction and "2" is in the direction transverse to the fibers.

The strains in the local system "1 – 2" are

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \frac{1}{2}\gamma_{12} \end{bmatrix} = \begin{bmatrix} 0{,}33 & 0{,}67 & \pm 0{,}94 \\ 0{,}67 & 0{,}33 & \mp 0{,}94 \\ \mp 0{,}47 & \pm 0{,}47 & -0{,}34 \end{bmatrix} \times \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \frac{1}{2}\gamma_{xy} \end{bmatrix}$$

The above equation is valid for angle-laminates with ± 55 degrees winding angles. The rotation of the strains from the global "x – y" system to the local "1 – 2" system gives

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \frac{1}{2}\gamma_{12} \end{bmatrix} = \begin{bmatrix} 0{,}33 & 0{,}67 & \pm 0{,}94 \\ 0{,}67 & 0{,}33 & \mp 0{,}94 \\ \mp 0{,}47 & \pm 0{,}47 & -0{,}34 \end{bmatrix} \times \begin{bmatrix} 1{,}07 & -0{,}44 & 0 \\ -0{,}44 & 0{,}61 & 0 \\ 0 & 0 & \frac{1}{2} \times 0{,}99 \end{bmatrix} \times \left[10^{-5}\right] \times \begin{bmatrix} \frac{P \times \Phi}{4 \times t} + \sigma_x \\ \frac{P \times \Phi}{2 \times t} \\ \frac{2 \times T}{\pi \times \Phi^2 \times t} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \frac{1}{2}\gamma_{12} \end{bmatrix} = \begin{bmatrix} 0{,}06 & 0{,}26 & \pm 0{,}47 \\ 0{,}57 & -0{,}09 & \mp 0{,}47 \\ \mp 0{,}71 & \pm 0{,}50 & -0{,}17 \end{bmatrix} \times \left[10^{-5}\right] \times \begin{bmatrix} \frac{P \times \Phi}{4 \times t} + \sigma_x \\ \frac{P \times \Phi}{2 \times t} \\ \frac{2 \times T}{\pi \times \Phi^2 \times t} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0{,}06 & 0{,}26 & \pm 0{,}47 \\ 0{,}57 & -0{,}09 & \mp 0{,}47 \\ \mp 1{,}42 & \pm 1{,}00 & -0{,}34 \end{bmatrix} \times \left[10^{-5}\right] \times \begin{bmatrix} \frac{P \times \Phi}{4 \times t} + \sigma_x \\ \frac{P \times \Phi}{2 \times t} \\ \frac{2 \times T}{\pi \times \Phi^2 \times t} \end{bmatrix} \qquad (5)$$

Equation (5) gives the strains in the local system "1 – 2" as a function of the external loading. We recall that in equation (5):

P is the fluid pressure.
Φ is the pipe diameter
t is the total wall thickness

σ_x is the external axial mean tensile stress
T is the applied external torque.

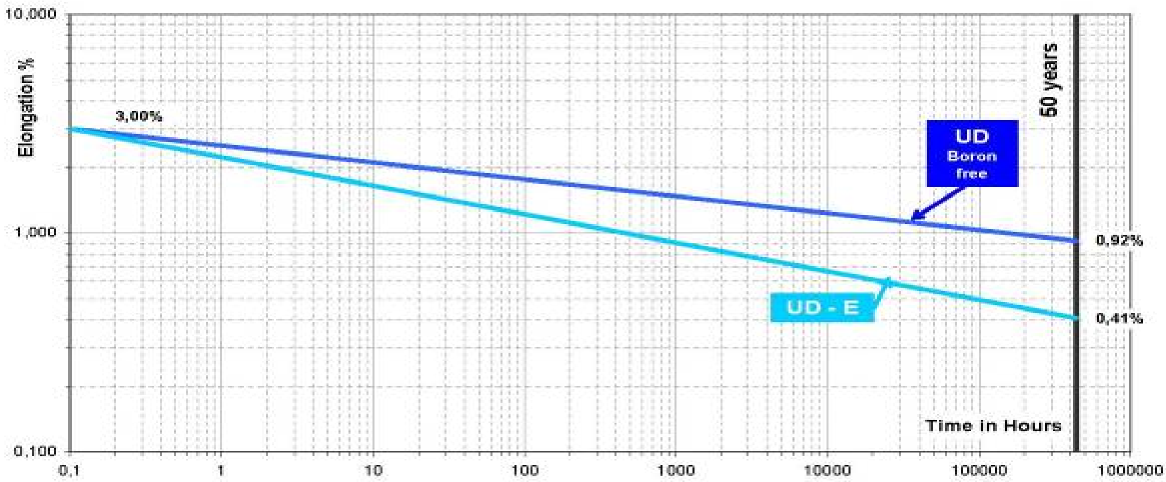
Equation (5) can solve **all structural problems related to ± 55 composite pipes**. Equation (5) is very easy to use, is accurate (in fact it is an exact equation) and does not require the definition of arbitrary and cumbersome failure envelopes.

We next comment on the failure strains

3.1 Failure strains: The following comments are applicable to failure strains referred to the local "1 – 2" ply system. There are three failure strains.

3.1.1 – Tensile strain in the fiber "1" direction. The tensile strain in the fiber direction controls the burst (rupture) failure of the pipe. In the short-term the pipe bursts when the tensile strain in the fiber direction reaches 3.0%. In the long-term the strain in the "1" direction that ruptures the fiber (and the pipe) depends on the interaction of the cyclic and the static loadings.

For pure static loadings the long-term rupture strain in the fiber direction varies with the chemical composition of the glass. Mark Greenwood has established that for a desired lifetime of 50 years, these strains are 0.92% for boron-free glass and 0.41% for regular E glass. For cyclic loadings the long-term strain varies with the number of cycles N and is independent of the glass composition.



Static regression line and long-term strain as reported by Mark Greenwood

The rupture failure of composite pipes caused by tensile strains in the fiber direction is not a concern for designers of composite pipes used in the oil and gas industry. In my opinion ISO 14692 does well in ignoring this mode of failure.

3.1.2 – Tensile strain in the transverse "2" direction. The "2" direction in composites parlance refers to the direction perpendicular to the glass fibers. The allowable transverse tensile strain is very small when compared to the strains allowable in the fiber "1" direction. The allowable transverse strains are determined from the glass-resin debonds that give rise to cracks along the fibers. The cracks caused by the transverse strains develop along the fibers and are characterized by three dimensions.

- The crack length “a” is determined by the pipe stiffness and the applied loading. Large fluid pressures produce long cracks. However, according to fracture mechanics, the crack length “a” is stationary under static loads. This is not the case for cyclic loads. Under cyclic loadings the length “a” is not stationary and the cracks increase in length with the number of cycles N.
- The crack opening “Δ” is also controlled by the applied loading and the pipe stiffness. For equal pressure loads the pipe having lower stiffness develops wider cracks. The crack opening “Δ” is stationary and does not grow even under cyclic loads. For any given load (cyclic or otherwise), the crack opening “Δ” is stationary as determined by the constant pressure and constant pipe stiffness.
- The crack spacing “δ” measures the number of cracks per unit length, or the crack density in the pipe wall. The crack density is a good measure of the damage that is done to the composite. Cyclic loadings are particularly damaging and, given enough cycles, can produce high crack densities.

The cracks we have just described are essential to the understanding of pipe failure. The pipe weeps when the water seeps through the pathways that are formed from these cracks. For the pathway to form, the crack length “a”, the opening “Δ” and the spacing “δ” must interact to form a passage for the water.

- Under static loads there is a “critical” strain below which the cracks do not coalesce and the pathways do not form. **We call this critical static strain the threshold weep strain.** Under static loads the cracks do not grow. And below the threshold strain, the cracks do not coalesce. Therefore, the **pipe never weeps** under static loads and below the threshold weep strain.
- Under cyclic loads the cracks maintain their openings, but grow in length and decrease in spacing. The crack openings are controlled by the pipe stiffness and do not change. Eventually, however, the crack growth from the cyclic loading will form the pathway for the water and the pipe will weep. There is no threshold weep strain for cyclic loads.

3.1.3 – Shear strain. The shear strains do not open cracks and do not cause weeping. The transverse tensile strains are the only strains that can open cracks and cause weeping. Composite pipes are not expected to weep in shear.

Table 1 summarizes the failure strains

Type of strain	Short-term rupture	50 years failure
Tensile in the 1 direction	3,00%	Boron-free glass 0,92% Regular E glass 0,41%
Tensile in the 2 direction (*)	0,25%	Depends on number of cycles
In-ply shear strain (*)	Not applicable	Not applicable

Table 1
Failure strains for composite pipes

(*) Note that the strain that weeps the pipe is the transverse tensile strain. The in-ply shear strain is not relevant for weep.

3.1.4 Strains from static loadings. The transverse tensile strain ϵ_2 controls the weep failure.

- If the transverse tensile strain is less than the threshold weep strain, the pathway will not form and the pipe will not weep.
- If the transverse tensile strain is above the threshold, the pathway will form and the pipe will weep. The weep time is the time taken by the water to travel through the tortuous pathway in the pipe wall. The weep time is actually a travel time.

The weep regression lines developed by the ASTM D 2992 B test protocol measure the time for the water to travel the stationary pathway formed by the coalesced cracks. This, of course, is true only if the transverse tensile strains ϵ_2 are larger than the threshold. **The weep time from the regression lines is nothing more than a travel time** and should not be interpreted as progressive deterioration of the pipe under static loads.

The travel time is not a basic material property. For one thing it varies with the pipe thickness. The travel time measured by the ASTM D 2992 B regression lines should not be used to design pipes. Instead, the pipe design should be based on the threshold weep strain.

3.1.5 Strains from cyclic loads. Under cyclic loads the transverse crack length "a" grows and the crack distance "δ" decreases until the cracks eventually coalesce and form the dreaded pathway. When that happens, the pipe starts the weep process. There is no threshold weep strain for cyclic loads. Given enough time, or enough cycles, all pipes under cyclic loads will weep.

The weep time for cyclic loadings is obtained by adding two times:

$$[weep \quad time] = [coalescence \quad time] + [travel \quad time]$$

- *Travel time.* The regression line developed in ASTM D 2992 A can predict the travel time for pipes with cracked plies. We say this with confidence, since the ASTM D 2992 A test protocol requires the application of test strains that are so high as to certainly crack the plies. The plies must be cracked before the water starts its travel. The travel time can be predicted using the ASTM D 2992 A regression lines.
- *Coalescence time.* The critical question to ask here is, how long will the tensile cyclic strain $\Delta\epsilon_2$ take to grow and coalesce transverse cracks in originally whole plies? We call this time the coalescence time. The coalescence time may be computed by the unified equation and is beyond the scope of this paper. The interested reader should refer to the Unified Equation.

The reader should keep in mind that no weeping will occur unless the UD ply is ruptured in the transverse direction. The transverse cracks may cause weep failure, but will not cause pipe rupture. Pipe rupture, as noted earlier, requires rupture of the fibers in the 1 direction.

4 Sample calculations. We next apply equation (5) and table 1 to several special cases.

4.1 *ASTM D 2105:* The external loading in this case consists of just the axial tensile axial stress. Therefore, from equation (5)

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0,06 & 0,26 & \pm 0,47 \\ 0,57 & -0,09 & \mp 0,47 \\ \mp 1,42 & \pm 1,00 & -0,34 \end{bmatrix} \times \left[10^{-5}\right] \times \begin{bmatrix} \frac{P \times \Phi}{4 \times t} + \sigma_x \\ \frac{P \times \Phi}{2 \times t} \\ \frac{2 \times T}{\pi \times \Phi^2 \times t} \end{bmatrix} \qquad (5)$$

By taking P = 0 and T = 0 we obtain

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0,06 & 0,26 & \pm 0,47 \\ 0,57 & -0,09 & \mp 0,47 \\ \mp 1,42 & \pm 1,00 & -0,34 \end{bmatrix} \times \left[10^{-5}\right] \times \begin{bmatrix} \sigma_x \\ 0 \\ 0 \end{bmatrix}$$

Expanding the above we obtain $\varepsilon_2 = 0,57 \times 10^{-5} \times \sigma_x$.

From table 1 we take €2 = 0.25% as the rupture transverse tensile strain. Therefore, the axial tensile stress that would fail the pipe in the short-term is

$$(\sigma)_x^{short-term} = \frac{1}{0,57 \times 10^{-5}} \times 0,0025 = 440 kg / cm^2$$

This is a very simple and direct calculation. The reader is invited to compare this approach with that from the failure envelop.

4.2 Split disc test: The loading in this case is the mean tensile hoop stress applied by the disc.

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0,06 & 0,26 & \pm 0,47 \\ 0,57 & -0,09 & \mp 0,47 \\ \mp 1,42 & \pm 1,00 & -0,34 \end{bmatrix} \times \left[10^{-5}\right] \times \begin{bmatrix} 0 \\ \sigma_y \\ 0 \end{bmatrix}$$

From table 1 we can take the short-term rupture strain in the fiber direction as 3.0%. From this we obtain

$$(\sigma)_y^{short-term} = \frac{1}{0,26 \times 10^{-5}} \times 0,03 = 11538 kg / cm^2$$ (tensile stress to rupture the glass fibers)

The split disc test applies only hoop loads to the specimen. Hoop loads do not fail the pipe in the 2 direction and therefore the split disc test cannot be used to measure the threshold weep strain.

4.3 Above ground pipes: The complete loading case for above ground pipes contemplates an applied tensile axial stress "σ_x", an internal pressure "P" and a torque "T". The complete equation, equation (5), is repeated below to facilitate the reading

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0{,}06 & 0{,}26 & \pm 0{,}47 \\ 0{,}57 & -0{,}09 & \mp 0{,}47 \\ \mp 1{,}42 & \pm 1{,}00 & -0{,}34 \end{bmatrix} \times \left[10^{-5}\right] \times \begin{bmatrix} \dfrac{P \times \Phi}{4 \times t} + \sigma_x \\ \dfrac{P \times \Phi}{2 \times t} \\ \dfrac{2 \times T}{\pi \times \Phi^2 \times t} \end{bmatrix} \qquad (5)$$

The above can be rearranged to give

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0{,}15 & 0{,}06 & \pm 0{,}30 \\ 0{,}10 & 0{,}57 & \mp 0{,}30 \\ \pm 0{,}15 & \mp 1{,}42 & -0{,}22 \end{bmatrix} \times \left[10^{-5}\right] \times \begin{bmatrix} \dfrac{P \times \Phi}{t} \\ \sigma_x \\ \dfrac{T}{\Phi^2 \times t} \end{bmatrix} \qquad (6)$$

4.4 Torque analysis: Equation (6) can be used to assess the effect of the torque "T". Expanding equation (6) we obtain

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0{,}15 \dfrac{P\Phi}{t} & + & 0{,}06\sigma_x & \pm & 0{,}30 \dfrac{T}{\Phi^2 t} \\ 0{,}10 \dfrac{P\Phi}{t} & + & 0{,}57\sigma_x & \mp & 0{,}30 \dfrac{T}{\Phi^2 t} \\ \pm 0{,}15 \dfrac{P\Phi}{t} & \mp & 1{,}42\sigma_x & - & 0{,}22 \dfrac{T}{\Phi^2 t} \end{bmatrix} \times \left[10^{-5}\right]$$

This equation can be used to compare the strains from the torque with the strains from the fluid pressure. By comparing these strains we can decide whether or not to ignore the torque.

Suppose we ignore torques producing strains that are less than 5.0% of the strains arising from the internal pressure P.

For strains in the fiber direction our criterion establishes that the torque should be ignored if

$$0{,}30 \frac{T}{\Phi^2 t} < 0{,}05 \times 0{,}15 \frac{P\Phi}{t}$$

$$T < 0{,}025 \times P\Phi^3$$

For strains transverse to the fibers our criterion gives

$$0{,}30 \frac{T}{\Phi^2 t} < 0{,}05 \times 0{,}10 \frac{P\Phi}{t}$$

$$T < 0{,}017 \times P\Phi^3$$

For shear strains our criterion gives

$$0{,}22\frac{T}{\Phi^2 t} < 0{,}05 \times 0{,}15\frac{P\Phi}{t}$$

$$T < 0{,}034 \times P\Phi^3$$

The transverse strains govern our choice and we can ignore torques less than

$$T < 0{,}017 \times P\Phi^3$$

5 Conclusion - The general equation (5) can be used to solve all possible loading scenarios found in practice. Equation (5) is very accurate and easy to use. And it requires no arbitrary definitions as those mentioned in the failure envelopes.